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ORIGINAL ARTICLE

A rule-extraction framework under multigranulation rough sets

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Abstract The multigranulation rough set (MGRS) is becoming a rising theory in rough set area, which offers a desirable theoretical method for problem solving under multigranulation environment. However, it is worth noticing that how to effectively extract decision rules in terms of multigranulation rough sets has not been more concerned. In order to address this issue, we firstly give a general rule-extraction framework through including granulation selection and granule selection in the context of MGRS. Then, two methods in the framework (i.e. a granulation selection method that employs a heuristic strategy for searching a minimal set of granular structures and a granule selection method constructed by an optimistic strategy for getting a set of granules with maximal covering property) are both presented. Finally, an experimental analysis shows the validity of the proposed rule-extraction framework in this paper.

keywords Multigranulation rough set · Rule extraction · Granulation selection · Granule selection

1 Introduction

Along with the development of information era, mass data have been collected and accumulated at a rapid pace. The useful information and knowledge hidden in large amounts of data are so much that we have an urgent need to mine potential rules and knowledge from rapidly growing data. For purpose of extracting implicit knowledge from the data, a great many of theories have been proposed and developed in recent years, which are fuzzy set theory [12], rough set theory [1, 2], computing with words [3, 13], granular computing [14, 16], computational theory for linguistic dynamic systems [4], and so on.

Rough set theory, introduced by Pawlak [1, 2], is a well-established mechanism for vagueness and uncertainty in data analysis. So for, it has been widely applied in knowledge discovery, decision analysis, pattern recognition and so on. In rough set theory, a target concept is always approximated by the so-called lower and upper approximations that are defined by an equivalence (indiscernibility) relation. According to various requirements, a great many of extensions of Pawlak's rough set have been developed, such as variable precision rough set [5], rough set based on tolerance relation [6, 7], Bayesian rough set [8], fuzzy rough set [9–11] and rough fuzzy set [9–11]. However, it can be seen that the above extensional rough sets are constructed on the basis of a single binary relation, which limits some applications of rough set theory.

Granular computing, proposed by Lin [14, 16], is an umbrella term that covers all theories, methodologies, techniques, and effective tools that uses granules in complex problem solving [17, 18]. With the granulation of universe, we consider elements within a granule as a whole rather than individually [15]. Elements in a granule are drawn together by indistinguishability, similarity, proximity, or functionality [14]. In the view of granular computing, Pawlak's rough set and most of the expanded rough sets are based on a single granulation generated from a binary relation, which is too restrictive for some practical

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applications. Therefore, Qian et al. [19, 20] took multiple granulations into account and proposed multigranulation rough sets. In multigranulation rough sets, a target concept can be described by multiple granulations instead of a single granulation in accordance with different requirements or targets of problem solving.

Since then, many researchers have extended the multigranulation rough set model so as to improve the modeling capability of classical multigranulation rough sets. Qian et al. [20] proposed a multigranulation rough set based on multiple tolerance relations in incomplete information systems. Lin et al. [21, 22] presented a covering-based pessimistic multigranulation rough set and also investigated a neighborhood-based multigranulation rough set. Xu et al. [24] put forward a variable precision multigranulation rough set. Yang et al. [25] investigated a multigranulation rough set based on fuzzy binary relations. Lin et al. [23] studied the multigranulation rough set theory via topology theory. However, these researches focused on generalizing the multigranulation rough set model and studying relative properties and applications. It is deserved to mention that Yang et al. [26] proposed the rules' measurements in terms of multigranulation rough sets, and the defined local and global measurements provided us theoretical basis for the definitions of reducts of MGRS. But it was not mentioned that how to extract decision rules effectively in the context of multigranulation rough sets. To address this issue, we introduce a rule-extraction framework including granulation selection and granule selection in terms of multigranulation rough sets. Because of using multigranulation view, the proposed rule-extraction framework in terms of MGRS is very desirable in many real applications, such as distributive information systems, multi-source information systems and data with high dimensions. In this paper, we just present a kind of solutions for the rule-extraction framework, i.e. a granulation selection method that employs a heuristic strategy and a granule selection method constructed by an optimistic strategy. It is worth noticing that weather the granulation selection or the granule selection are both based on the model of multigranulation rough sets and keep the positive region in MGRS unchanged.

The paper is organized as follows. To facilitate our discussions, we first introduce some basic concepts of multigranulation rough sets in Sect. 2 Follow on to Sect. 3, a general rule-extraction framework including granulation selection and granule selection in terms of multigranulation rough sets is developed. Then a granulation selection algorithm is presented, which employs a heuristic strategy for searching a minimal set of granular structures. And a granule selection algorithm is also developed, which is constructed by an optimistic strategy for getting a set of granules with maximal covering property. Finally, experimental results are summarized in Sect. 4.

2 Preliminaries

In this section, we review some basic concepts of multigranulation rough sets.

2.1 Multigranulation rough sets

Multigranulation rough set, proposed by Qian et al. [19, 20], is a new extension of rough set. In multigranulation rough sets, a target concept can be approximated by multiple granulations instead of a single granulation according to different requirements or targets of problem solving.

Definition 1 Let $S=(U,V,A,f)$ be an information system, $A=C\cup D$, and $C\cap D=\varnothing$, then S is called a decision information system, attributes in C are called condition attributes and attributes in D are called decision attributes.

Definition 2 [19] Let $S=(U,V,A,f)$ be a decision information system, $A=C\cup D, X\subseteq U$ and $A_1,A_2,\ldots,A_m\subseteq C$, then the lower approximation and the upper approximation of X related to $A_1, A_2,\ldots, A_m$ in terms of multigranulation rough set are defined as:

$$\underline{\sum_{i=1}^{m}}A_i(X)=\{x\in U:[x]_{A_1}\subseteq X\vee[x]_{A_2}\subseteq X\vee\ldots\vee[x]_{A_m}\subseteq X\},$$

$$\overline{\sum_{i=1}^{m}}A_i(X)=\sim(\underline{\sum_{i=1}^{m}}A_i(\sim X)).$$

The area of uncertainty boundary region in MGRS can be extended as:

$$Bn_{\sum_{i=1}^{m}A_i(X)}=\overline{\sum_{i=1}^{m}}A_i(X)-\underline{\sum_{i=1}^{m}}A_i(X).$$

Definition 3 [19] Let $S=(U,V,A,f)$ be a decision information system, $A=C\cup D,A_1,A_2,\ldots,A_m\subseteq C$, and U/D a partition of the decision attribute D. The approximation quality of D by $A_1, A_2,\ldots, A_m$ (i.e. the degree of dependence) is defined as:

$$\gamma_{\sum_{i=1}^{m}A_i}(D)=\frac{\sum\{|\underline{\sum_{i=1}^{m}}A_i(Y)|:Y\in U/D\}}{|U|}.$$

2.2 Multigranulation construction of information systems

For information systems, each granular structure can be induced by a single attribute, or a family of attributes. In this paper, we only discuss a case of multigranulation construction, i.e. each granular structure is generated by the equivalence relation induced by one attribute. On the basis of this premise, we give the definition of granular structure and the notation of granule in order to facilitate the following discussion.

Definition 4 Let $S = (U, V, A, f)$ be a decision information system, $A_i \subseteq A$, and R_i the equivalence relation induced by A_i, then the granular structure of A_i is defined as $GS_{A_i} = U/R_i = \{[x]_{R_i} | x \in U\}$.

Meanwhile the granule containing x in the granular structure GS_{Ai} is denoted as:

$g_{A_i}(x) = [x]_{R_i}, x \in U.$

In fact, the above view is the simple case of multigranulation construction. In many real applications, each granular structure in information systems also can be induced by a family of attributes, not but a single attribute. In that case, the definitions of granular structure and granule need to be further studied.

Example 1 Here, we employ an example to show the granular structures and granules in an information system. Table 1 depicts an information system about emporium investment project. Locus, investment and population density are condition attributes. And decision stands for decision attribute. (In the sequel, L, I, P and D will stand for locus, investment, population density and decision, respectively.)

In this information system, we can get that the granular structure of P is $GS_P = \{\{e1, e2\}, \{e3, e4, e5\}, \{e6, e7, e8\}\}$, and the granule which contains $e1$ in granular structure GS_P is $g_P(e1) = \{e1, e2\}$.

3 Rule extraction in multigranulation rough sets

In this section, a general rule-extraction framework including granulation selection and granule selection in terms of multigranulation rough sets is first proposed. Then we present two kinds of algorithms under the rule-extraction framework, which are a heuristic granulation selection algorithm for removing redundant granular structures and a granule selection algorithm constructed by an optimistic strategy for getting a set of granules with maximal covering property. Finally, an experimental analysis is performed to show the validity of the proposed rule-extraction framework in terms of MGRS.

Table 1 An information system about emporium investment project

Project	Locus	Investment	Population density	Decision
e1	Common	High	Big	Yes
e2	Bad	High	Big	Yes
e3	Bad	Low	Small	No
e4	Bad	Low	Small	No
e5	Bad	Low	Small	No
e6	Bad	High	Medium	Yes
e7	Bad	High	Medium	No
e8	Good	High	Medium	Yes

3.1 A general rule-extraction framework

It is a nontrivial task to extract rules from data sets. And there are a large number of methods for rule extraction in terms of rough set. However, these methods are based on a single granulation, which is too restrictive for some practical applications. Therefore, we introduce a rule-extraction framework in the context of multigranulation rough sets, in which a target concept can be approximated by multiple granulations according to different requirements or targets of problem solving. From the viewpoint of rough set's applications, the proposed rule-extraction framework in terms of MGRS is very desirable in many real applications, such as distributive information systems, multi-source information systems and data with high dimensions.

Intuitively, for information systems, there exist a great many of granular structures, but not all of them are absolutely necessary for the concept approximation in terms of multigranulation rough sets. If some granular structures are not significant, it is our responsibility to get rid of them from further consideration. Therefore, granulation selection is put forward to remove relatively redundant granular structures. After the granulation selection, we get a reduced set of granular structures, which contains a number of granules. All of these granules compose a set of granules. However, several granules in the granule set may be redundant and their removal has little impact on extracting rules efficiently. Thus, granule selection is proposed to get a reduced set of granules. Then through the intension of these reserved information granules, we can extract decision rules with good generalization [1]. From the above, it is clear to see that the rule-extraction framework in terms of MGRS mainly includes two parts, i.e. granulation selection and granule selection. The following Fig. 1. shows the whole process of the rule-extraction framework including granulation selection and granule selection in the context of multigranulation rough sets.

[1] The generalization is an important index of depicting classifier performance. In our study, we only discuss the lower approximation reduction. If we want to analyze the generalization of extracted rules in detail, we should take the upper approximation into account and design the corresponding multigranulation rough classifier. However, describing these contents in detail is beyond the scope of this paper. We will focus on the studies of the multigranulation rough classifier and its generalization in the future work.

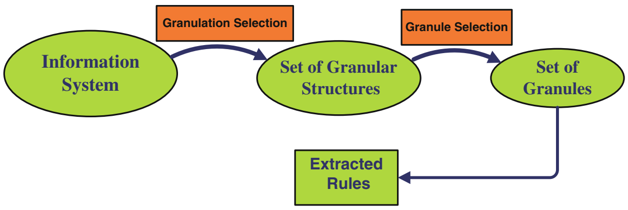


Fig. 1 Rule extraction process of the general framework in terms of MGRS

3.2 A heuristic granulation selection algorithm

As known to all, there are a large number of granulation selection methods for information systems in accordance with different granulation constructions in terms of multi-granulation rough sets. Even for an information system with certain multigranulation construction, the solution for granulation selection can be diverse according to different search strategies and evaluating measures.

In this paper, we discuss the simple multigranulation construction for information systems, i.e. each granular structure is induced by one attribute. And on the basis of this, we put forward a kind of granulation selection method in terms of MGRS, which is a granulation selection algorithm that employs a heuristic strategy. By using this heuristic granulation selection algorithm, we want to remove several relatively insignificant granular structures, and then get a minimal set of granular structures for further consideration.

In what follows, we firstly give the definition of granulation selection for information systems in the context of multigranualtion rough sets.

Definition 5 Let $S = (U, V, A, f)$ be a decision information system, $A = C \cup D, B \subseteq C$. If $\gamma_B(D) = \gamma_C(D)$ and there exists no $B' \subset B$ such that $\gamma_{B'}(D) = \gamma_C(D)$, then the set of granular structures B is regarded as a granulation selection result of S. [2]

In order to use a heuristic strategy for constructing a granulation selection algorithm, we need to introduce two heuristic functions.

Definition 6 Let $S = (U, V, A, f)$ be a decision information system, $A = C \cup D, B \subseteq C. \forall a \in B$, the inner importance of granular structure a is defined as:

$$GSSIG_{in}(a, B, D) = \gamma_B(D) - \gamma_{B-\{a\}}(D),$$

and $\forall a \in C - B$, the outer importance of granular structure a is defined as:

$$GSSIG_{out}(a, B, D) = \gamma_{B\cup\{a\}}(D) - \gamma_B(D).$$

Theorem 1 From the definition of inner importance of granular structure, we can draw the conclusion that $GSSIG_{in}(a, B, D)$ has the following properties:

(1) $0 \leq GSSIG_{in}(a, B, D) \leq 1$;
(2) $\forall a \in C$, if $GSSIG_{in}(a, B, D) > 0$, the granular structure a is necessary.

Definition 7 Let $S = (U, V, A, f)$ be a decision information system, $A = C \cup D, B \subseteq C$. B is called as a core set of granular structures if and only if $\forall a \in B$, $GSSIG_{in}(a, B, D) > 0$.

For simplicity, the core set of granular structures is written as

$$GSCORE_B = \{a | GSSIG_{in}(a, B, D) > 0, a \in B\}.$$

Based on the above introduction, we present a heuristic granulation selection algorithm in terms of MGRS for information systems. In this granulation selection approach, two important measures of granular structures are used for heuristic functions, i.e. inner importance measure and outer importance measure. To determine the significance of granular structures, the inner importance measure is put into use; while the outer importance measure is used in forward granulation selection. In the forward greedy granulation selection approach, starting with the core set of granular structures, we put the granular structure with the maximal outer significance into the subset of granulation structures in each step until the subset of granulation structures satisfies the stopping criterion, and then we call the subset of granulation structures as a minimal set of granular structures. Formally, the heuristic granulation selection algorithm can be written as follows.

Algorithm 1. A heuristic granulation selection algorithm
Input: Decision information system $S = (U, C \cup D)$
Output: The minimal set of granular structures $S: GS$

1. $GS \leftarrow \emptyset, GSCORE \leftarrow \emptyset$;
2. Calculate $GSSIG_{in}(a_i, B, D), i \leqslant |C|$;
   and if $GSSIG_{in}(a_i, B, D) > 0$, put a_i into $GSCORE$;
3. If $GSCORE = \emptyset$, go to step 4 ; Otherwise, go to step 5;
4. $B \leftarrow GS$;
   If $a_j \in (C - B)$, calculate $GSSIG_{out}(a_j, B, D)$,
   $GSSIG_{out}(a_{j_{max}}, B, D) = max\{GSSIG_{out}(a_j, B, D), a_j \in (C - B), 1 \leq j \leq |C|\}$;
   $B \leftarrow B \cup \{a_{j_{max}}\}$, go to step 6;
5. If $\gamma_{GSCORE}(D) = \gamma_C(D)$, $GS \leftarrow GSCORE$, go to step 7;
   Otherwise, $B \leftarrow GSCORE$, go to step 7;
6. If $\gamma_{GSCORE}(D) = \gamma_C(D)$, $GS \leftarrow B$, go to step 7; Otherwise, go to step 4;
7. Return GS.

[2] The granulation selection in terms of MGRS is based on the model of multigranulation rough sets, which keeps the positive region in MGRS unchanged (i.e. the definition of approximation quality is based on multigranulation rough sets theory). The proposed granulation selection is different from the attribute reduction in terms of rough set.

The time complexity of computing all the importance of granular structures in an information system is $O(|C||U|^3)$. In following steps, begin with the core set of granular structures, we add the granular structure with the maximal outer importance into the set of granulation structures in each loop until the set of granulation structures satisfies the stopping criterion. The time complexity of these steps is $O(|C|\log|C| + |C||U|^3) = O(|C||U|^3)$. Therefore, the entire time complexity of Algorithm 1 is $O(|C||U|^3)$.

Example 2 Through using Algorithm 1, we get a minimal set of granular structures of the information system mentioned in Example 1. The reduced set of granular structures is $GS = \{$ *Locus*, *Population density* $\}$, which is shown in Table 2.

In Table 1, the granular structure of L is $\{\{e1\}, \{e2, e3, e4, e5, e6, e7\}, \{e8\}\}$,
the granular structure of I is $\{\{e1, e2, e6, e7, e8\}, \{e3, e4, e5\}\}$,
and the granular structure of P is $\{\{e1, e2\}, \{e3, e4, e5\}, \{e6, e7, e8\}\}$.
The granular structure of D is $\{\{e1, e2, e6, e8\}, \{e3, e4, e5, e7\}\}$. Let $X_1 = \{e1, e2, e6, e8\}$, and $X_2 = \{e3, e4, e5, e7\}$.
In the context of multigranulation rough sets, the lower approximation of X_1 related to granular structures L, I and P is $\underline{X_1}_{L+I+P} = \{e1, e2, e8\}$.
And the lower approximation of X_2 related to granular structures L, I and P is $\underline{X_2}_{L+I+P} = \{e3, e4, e5\}$.
By computing the importance of each granular structure in the information system, we get the core set of granular structures that is $\{$ *Locus*, *Population density* $\}$. Because this set of granulation structures satisfies the stopping criterion $\gamma_{\{L, P\}}(D) = \gamma_{\{L, I, P\}}(D)$, the final granulation selection result is $\{$ *Locus*, *Population density* $\}$.
Through the intension of the granules(i.e. $\{e1\}, \{e8\}, \{e1, e2\}, \{e3, e4, e5\}$) in granular structures L and P, one can make "*OR*" rules as follows: $(L = Common) \vee (P = Big) \Rightarrow (D = Yes)$,
$(L = Good) \vee (P = Big) \Rightarrow (D = Yes)$,
$(P = Small) \Rightarrow (D = No)$.

3.3 A granule selection algorithm constructed by an optimistic strategy

After the granulation selection, we get a minimal set of granular structures, which contains a number of granules. All of these granules compose a set of granules. However, some granules in the granule set may be redundant, and their removal has little impact on extracting rules efficiently. Therefore, we present a granule selection method to remove several granules in the granule set and get a reduced set of granules with maximal covering property, then we call this as an optimistic strategy.

In what follows, we give the definition of granule selection for a certain set of granules.

Let $Ug = \{x_1, x_2, \ldots, x_p\}$ be the universe of discourse, and $Cg = \{g_1, g_2, \ldots, g_q\}$ a family of granules of Ug that meets $\bigcup_{i=1}^{q} g_i = Ug$, then Cg is a covering of Ug [28].

Definition 8 Let $Ug = \{x_1, x_2, \ldots, x_p\}$ be the universe of discourse, $Cg = \{g_1, g_2, \ldots, g_q\}$ a covering of $Ug, g \subset Ug$ and $g_i \in Cg$. If $\exists g_j \in Cg$, such that $g_i \subset g_j \subset g$, we say g_i is a relatively reducible granule of Cg with respect to g; otherwise, g_i is relatively irreducible.

Definition 9 Let $Ug = \{x_1, x_2, \ldots, x_p\}$ be the universe of discourse, $Cg = \{g_1, g_2, \ldots, g_q\}$ a covering of Ug, and $Cg' \subseteq Cg$. If $\forall g_i \in Cg', g_i$ is relatively irreducible, then Cg' is relatively irreducible, and Cg' is regarded as a granule selection result of C.

Remark 1 Through using the proposed optimistic granule selection, we retain much bigger granules in the granule set and get a reduced set of granules with maximal covering property. Furthermore, the classifier constructed by the extracted decision rules can possess much better generalization. we take our ideas from the relative covering reduction [30, 31], which is different from covering reduction [27–29].

In what follows, a granule selection algorithm constructed by an optimistic strategy in terms of multigranulation rough sets is presented. In this granule selection algorithm, starting with the set of granules generated from the minimal set of granular structures, we remove the relatively reducible granules from the granule set step by step until all the granules in the reduced set of granules are relatively irreducible. Formally, the granule selection algorithm constructed by an optimistic strategy can be written as follows.

Algorithm 2. A granule selection algorithm using an optimistic strategy.

Input: A covering of Ug ($i.e. Cg = \{g_1, g_2, ..., g_q\}$)
Output: A granule selection result of Cg: G

1. $G \leftarrow Cg$;
2. $\forall g_i \in Cg$, if $\exists g_j \in Cg$, such that $g_i \subset g_j$, then $G \leftarrow G - g_i$;
3. Return G.

The time complexity of Algorithm 2 (i.e. a granule selection algorithm using an optimistic strategy) is $O(|CS|^2|U|^2)$.

Example 3 Continued from Example 2, by using the granule selection algorithm constructed by an optimistic strategy, we get a result of granule selection that is $\{\{e8\}, \{e1, e2\}, \{e3, e4, e5\}\}$.

Table 2 A set of granular structures of the information system in Example 1

Project	Locus	Population density	Decision
e1	Common	Big	Yes
e2	Bad	Big	Yes
e3	Bad	Small	No
e4	Bad	Small	No
e5	Bad	Small	No
e6	Bad	Medium	Yes
e7	Bad	Medium	No
e8	Good	Medium	Yes

After the granulation selection, we get a set of granules generated from the minimal set of granular structures, which is $\{\{e1\}, \{e8\}, \{e1, e2\}, \{e3, e4, e5\}\}$. Because the granule $\{e1\}$ is relatively reducible, $\{e1\}$ is removed from $\{\{e1\}, \{e8\}, \{e1, e2\}, \{e3, e4, e5\}\}$. For the reason that all the granules in the reduced granule set are relatively irreducible, the final result of granule selection is $\{\{e8\}, \{e1, e2\}, \{e3, e4, e5\}\}$.

Through the intension of the information granules in the reduced set of granules, one can extract certain "*OR*" rules, i.e.

$(L = Good) \vee (P = Big) \Rightarrow (D = Yes)$,
$(P = Big) \Rightarrow (D = Yes)$,
$(P = Small) \Rightarrow (D = No)$.

3.4 Experimental analysis

In the empirical study, five data sets from the University of California at Irvine Machine Learning Repository are used, and the information about these data sets is shown in Table 3. The experiments are designed to show that the proposed rule-extraction framework including granulation selection and granule selection in terms of MGRS is reasonable and effective.

The proposed rule-extraction framework in terms of multigranulation rough sets has a potential application on real data. There are three main attribute types of real data, which are categorical, numerical and mixed. For categorical data, the proposed rule-extraction framework can be applied directly to dealing with it. For numerical data, the data set can be discretized as the corresponding categorical data set, and then can be analyzed by the rule-extraction framework in terms of MGRS. For mixed data, we can use the rule-extraction framework in the context of multigranulation rough fuzzy sets in order to deal with it. For these more complex data sets, they can be analyzed by the same framework in terms of MGRS, we omit the relative experimental analysis here.

Table 3 Data description

	Data sets	Objects	Granular structures	Classes
1	Table 1	8	4	2
2	Tae	151	6	3
3	Zoo	101	17	7
4	Glass	214	10	6
5	Transfusion	748	5	2

In this section, in order to emphasize the validity of the proposed rule-extraction framework including granulation selection and granule selection in terms of MGRS, two methods are employed for the comparison analysis, which are the attribute reduction method based on attributes significance in terms of rough set and the method of relative covering reduction.

Figure 2 shows the variation of granular structures' number in five data sets through using the heuristic granulation selection method under the rule-extraction framework in terms of MGRS, which is compared with the attribute reduction method based on attributes significance in terms of rough set. In Fig. 2, blue polygonal line stands for the number of granular structures in original data sets, green polygonal line stands for the number of granular structures reduced by the attribute reduction method based on attributes significance in terms of rough set, and orange polygonal line stands for the number of granular structures reduced by the heuristic granulation selection method under the rule-extraction framework in terms of MGRS. From this chart, we can draw a conclusion that the number of granular structures in information systems can be reduced by both of the two methods. However, the heuristic granulation selection method under the rule-extraction framework in terms of MGRS is different from the attribute reduction method based on attributes significance in terms of rough set.

In Fig. 3, the comparison of extracted rules and consuming time with the proposed rule-extraction framework and the relative covering reduction method are both

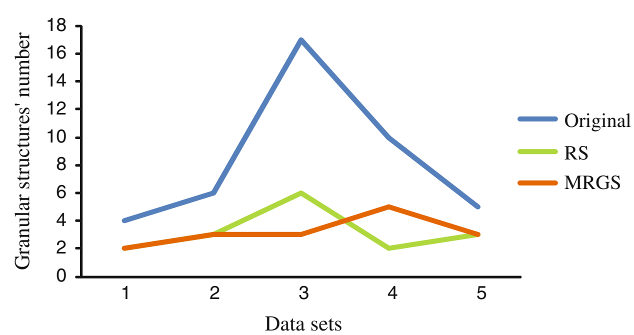
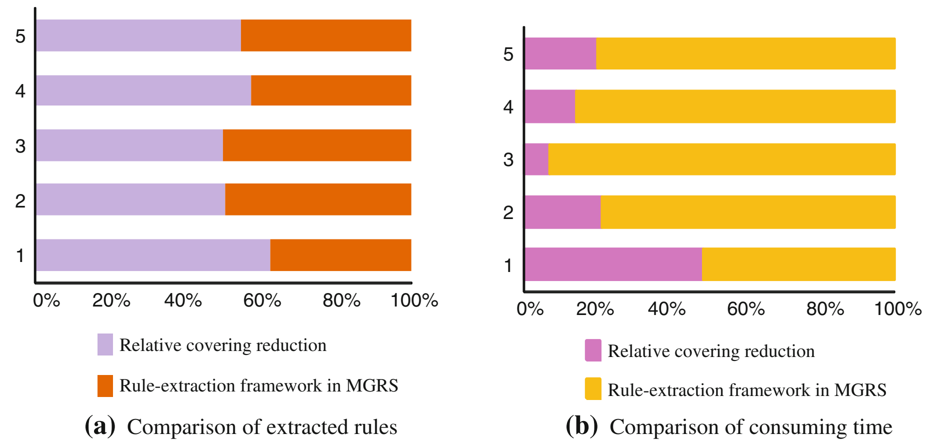


Fig. 2 Variation of granular structures

Fig. 3 Comparison between the rule-extraction framework in MGRS and relative covering reduction



displayed. In subfigure (a), for each data set, the orange bar stands for the number of extracted rules through using the proposed rule-extraction framework, and the purple bar states the number of extracted rules by using the relative covering reduction method. From chart (a), it can be seen that the number of extracted rules can be reduced through using the rule-extraction framework in terms of MGRS. Furthermore, the classifier constructed by the extracted rules can possess much better generalization. In subfigure (b), for each data set, the yellow bar stands for the proportion of consuming time using the rule-extraction framework in terms of MGRS, and the dark purple bar stands for the proportion of consuming time using the relative covering reduction method. From chart (b), we can find that the proposed methods for the rule-extraction framework in terms of MGRS (i.e. the heuristic granulation selection algorithm and the granule selection algorithm constructed by an optimistic strategy) are time consuming.

From Fig. 2, Fig. 3 and the above analysis, we draw a conclusion that through using the rule-extraction framework in terms of MGRS, the number of granular structures and granules in information systems can be largely reduced, and the multigranulation classifier constructed by the extracted rules can possess much better generalization. It's worth noticing that as a good solution for the rule-extraction framework in terms of multigranulation rough sets, the methods (i.e. the heuristic granulation selection algorithm and the granule selection algorithm constructed by an optimistic strategy) are time consuming.

In this study, from the analysis of the entire paper, it can be seen that each granular structure is induced by one attribute in information systems. In fact, this view is the simple case of multigranulation construction. In many real applications, each granular structure also can be induced by a family of attributes, not but a single attribute. In that case, granulation selection under the rule-extraction framework in terms of MGRS is not just a problem of granulation selection for information systems, but a problem including granulation selection for each granular structure and granulation selection for information systems. In further work, we will address how to extract decision rules and learn a classifier via every granular structure induced by a set of attributes in the context of multigranulation rough sets.

4 Conclusions

Rule extraction is a valuable research issue in the context of multigranulation rough sets. To address this issue, through including granulation selection and granule selection, a general rule-extraction framework in terms of MGRS has been proposed. In this framework, a heuristic granulation selection algorithm has first been established, which can be used to get a minimal set of granular structures. And then a granule selection algorithm constructed by an optimistic strategy has also been given, which can be used to get a set of granules with maximal covering property. Through the intension of these reserved information granules, we can extract decision rules with good generalization. The experimental analysis has shown that the rule-extraction framework in terms of multigranulation rough sets in this paper is reasonable and effective.

It is deserved to point out that the granulation selection and granule selection in the rule-extraction framework are both motivated by a kind of multigranulation view. The rule-extraction framework in terms of MGRS will have a number of potential practical applications in distributive information systems, multi-source information systems, data with high dimensions, and so on.

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